

# SU(2) Relativity and the EPR Paradox

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## Abstract

In the normal presentation of the EPR problem a comparison is made between the (weak) Copenhagen interpretation of quantum mechanics which seems to suggest that at times action at a distance may take place, and the hidden parameter interpretation which must satisfy Bell's inequality, in contradiction to the predictions of quantum mechanics. In this paper, we consider a relativistic approach to the paradox. However, the frame of reference under consideration is not the usual Lorenz frame but rather the spin frame of reference which is invariant with respect to the SU(2) group.

Key WORDS: hidden variables, Copenhagen, SU(2) relativity

## 1 INTRODUCTION

Before giving an alternative approach to the EPR problem, we must first be clear as to what the paradox is and what a proper solution should entail: Consider two particles emitted in the singlet state. Let  $\lambda_1$  be the measurement of spin made on particle 1 in the direction  $\vec{n}_1$ . By definition of a singlet state, a measurement of the spin of particle 2 will yield the value  $\lambda_1$ , if measured in the same direction  $\vec{n}_1$ , a fact also observed in experiment.

The paradox arises when two competing epistemological views of reality are brought to bear on the matter. In the naive realist interpretation [10] (as it has come to be called) the particles have a definite predetermined spin value. From this perspective, reality can be known by simply "looking" at it [6]. What we see is really what is there, and moreover, based on these assumptions, a naive realist can construct a mathematical model of spin which gives rise either to Bell's inequality [3] or the GHZ theorem [9]. In either case, a mathematical contradiction arises which cannot be explained by naive realism.

The weak Copenhagen interpretation of QM, on the other hand, also runs into a difficulty. At the core of this interpretation is the projection postulate which effectively says that the act of measurement forces the particle to choose

between the two alternatives. However, if this is the case, it means that the particles in the singlet state choose equal but opposite values on measurement, although communication between the particles has been eliminated. This state of affairs is normally explained by saying that the interaction is “non-local” which is equivalent to saying that there is action at a distance, and some perhaps would interpret this to mean that cause and effect have broken down.

Therein lies the paradox. Naive realism gives rise to a contradiction while the projection postulate, if taken literally gives rise to action at a distance, which appears to be a violation of relativity theory.

The epistemological approach taken here is to view the spin state as preexisting while at the same time considering the measured value of the spin as an  $SU(2)$  relativity effect correlated to the direction of measurement, a fact previously noted in other publications [5, 7]. In other words, the same spin state can be interpreted as +1 or -1 depending on our point of view. Consequently, as we shall see below, this permits two different values to be assigned to the same event, and will result in Bell’s inequality, if the  $SU(2)$  relativity effect is ignored.

An analogy might help. The Earth as viewed from the North pole can be seen as rotating in an anti-clockwise direction (-1), while the same state of motion can be viewed as being in a clockwise direction (+1), if viewed from the South pole. However, in the notation of quantum mechanics this same event, as seen from the equator, cannot be interpreted in a consistent way and it is this inconsistency that gives rise to both Bell’s inequality and the GHZ result. In other words, the same state can take on two different values, depending on the viewpoint and since the essence of Bell’s inequality involves correlations from three different directions, it’s a prime candidate for bringing forth inconsistencies, unless this  $SU(2)$  relativity effect is taken into account [5]. Another useful analogy is to envisage a Mobius strip and ask what is the orientation of the surface? It has no orientation, although from any one perspective it can be considered a “two-faced” surface. Once again, orientation depends on the point of view [7].

## 2 DIFFERENT REPRESENTATIONS

The first thing to grasp is that there are two equal but different representations of spin. Specifically, consider the following:

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \end{pmatrix} \quad (1)$$

which means that

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = - \begin{pmatrix} -1 \\ 0 \end{pmatrix}. \quad (2)$$

It is clear that if the ket  $|s\rangle$  represents the spin state and  $U \in SU(2)$  then  $\langle s|U^*U|s\rangle$  is invariant regardless of the representation. This is what we mean by  $SU(2)$  spin invariance. We now separately analyze each representation. Following the notation of Greenberger et al [9], we can write  $|\vec{n}, +\rangle$  and  $|\vec{n}, -\rangle$  to

represent spin-up and spin-down respectively along the  $\vec{n}$  direction. Therefore,

$$|\vec{n}_1, +\rangle = (\cos \theta/2) |\vec{n}_2, +\rangle + (\sin \theta/2) |\vec{n}_2, -\rangle. \quad (3)$$

Now choose  $\vec{n}_2$  by rotating through  $\theta = \pi/2$  (clockwise) with respect to  $\vec{n}_1$  and choose  $\vec{n}_3$  by rotating through  $\theta = -\pi/2$  (anti-clockwise) with respect to  $\vec{n}_1$  (Fig. 1). Then equation (3) gives

$$|\vec{n}_1, +\rangle = 1/\sqrt{2} |\vec{n}_2, +\rangle + 1/\sqrt{2} |\vec{n}_2, -\rangle \quad (4)$$

$$= 1/\sqrt{2} |\vec{n}_3, +\rangle - 1/\sqrt{2} |\vec{n}_3, -\rangle. \quad (5)$$

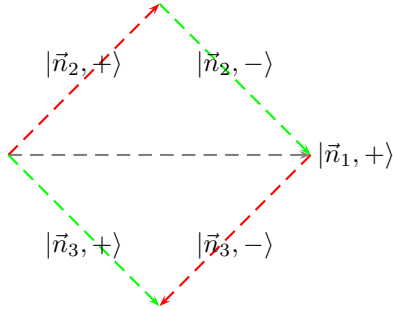


Fig. 1

In particular if  $|\vec{n}_1, +\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ , then  $|\vec{n}_2, +\rangle = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$  and  $|\vec{n}_2, -\rangle = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$ , and  $|\vec{n}_3, +\rangle = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$  and  $|\vec{n}_3, -\rangle = \begin{pmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$ . Note, immediately that  $\langle \vec{n}_2, \pm | \vec{n}_3, \pm \rangle = 0$  and  $|\vec{n}_2, -\rangle = |\vec{n}_3, +\rangle$ . This creates the ambivalent situation of identifying, from the perspective of  $\vec{n}_1$ , a spin-up state ( $|\vec{n}_2, -\rangle$ ) with a spin-down state ( $|\vec{n}_3, +\rangle$ ), which forces the obvious question, as to what is the meaning of up or down in this case. In fact from the perspective of  $\vec{n}_1$  alone there is no consistent meaning; for the value depends not only on the direction  $\vec{n}_1$  but on the choice of angle  $\theta$  defined relative to  $\vec{n}_1$ . It is precisely this ambivalence that is at the heart of the GHZ inconsistency and Bell's inequality.

### 3 COPENHAGEN OR REALISM

In the light of the above relativistic approach, we now ask how the above process can be viewed from both the perspective of Bell's original paper on hidden variables [3] and from the perspective of the (weak) Copenhagen interpretation. Bell assumes that "if the two measurements are made at places remote from one another the orientation of one magnet does not influence the result obtained with the other" (II Formulation), and suggests that if this "locality" hypothesis fails then "the statistical predictions of quantum mechanics are incompatible with

separable, [local] predetermination” (V. Generalization). This calls for some comments and observations.

(1) The word “influence” in the first quote can have two meanings. One meaning can refer to a physical “influence” between the two magnets, typified by some type of physical communication as in the case of an electromagnetic transmission. The other “influence” can come from a type of knowledge associated with the rules of conditional probability and SU(2) relativity.

(2) In this paper, the term “influence” is only been used in the second sense, which means we are assuming no physical contact between the magnets and the Lorentz invariance is always maintained with respect to space-time.

(3) The second type of influence allows for instantaneous knowledge of physical events beyond the light cone, without in anyway violating the laws of physics or causality. To understand this better, let us consider the following analogous example. Alice and Bob are each given a sealed envelope containing one of two possible cards. Alice’s envelope contains the Ace of Hearts, while Bob’s envelope contains the Ace of Spades. Each one of them is sent on a long train journey going in opposite directions and are told to open their envelopes after one hour. If Alice were asked, prior to opening her envelope, the probability that Bob had the Ace of Hearts she would answer  $1/2$ . However, if she were asked the same questions, after she had opened her envelope, she would have responded 0. In other words, by Alice opening her first envelope, her new found knowledge “influences” the probability outcome of Bob’s experiment. Knowledge, of the content of her envelope gives her instantaneous knowledge of Bob’s envelope. However, this “influence” is grounded in the rules of conditional probability and the law of large numbers, without in any way violating the rules of causality. Alice is not free to communicate her instantaneous knowledge to Bob in an instantaneous way. Communication is subjected to Lorentz invariance.

Similarly, in an analogous way, when one component of spin is measured in a singlet state, we not only instantaneously know the other but spin values can be preassigned by nature prior to the experiment, although not necessarily in any deterministic way. Moreover, precisely because of the SU(2) relativity effect (as described previously), preassigned eigenstates (in a direction  $\vec{n}_1$ ) cannot be used to mathematically determine the preassigned eigenstates of another direction,  $\vec{n}_2$ . Hence, our inability to determine the values of preassigned states, is in full agreement with Bell’s assertion that “the statistical predictions of quantum mechanics are incompatible with separable predetermination” of observed values by means of a mathematical formula. However, it would not be in agreement with the assertion that “the statistical predictions of quantum mechanics are incompatible with separable predetermination” in an ontological sense. In other words, nature obeys the rules of causality but not necessarily in any systematic way that precludes chance, nor in any absolute way to preclude relativity by determining absolute values.

(4) Bell has rightly assumed that “non-locality” implies “no separable predetermination” of spin values by means of a mathematical formula based on hidden parameters. However, he also seems to imply a fallacy, when he concludes the opposite, namely that a theory which involves “no separable predetermination”

of spin values could not be “Lorentz invariant”. In contrast, this paper has pointed out, that SU(2) relativity implies that there is no separable predetermination of measurable spin values by mathematical means but yet the theory is still local and hence Lorentz invariant. This means that for any arbitrarily chosen direction, the spin values are preassigned prior to the experiment. However, precisely because of the SU(2) relativity effect (as described above), preassigned eigenstates in one direction ( $\vec{n}_1$ ) cannot be used to determine preassigned eigenstates in another ( $\vec{n}_2$ ). Nevertheless, cause and effect are preserved but not systematically. The rules of chance always apply, and values cannot be assigned to spin in an absolute way, for the same reason absolute mass cannot be assigned to each of the particles. Finally, note that the SU(2) frame of reference allows us to determine not the actual outcome of the experiment but rather the probability distribution for the spin-observables of the pre-correlated states.

(5) The Copenhagen interpretation will also undergo modifications, depending on how one interprets the projection postulate. To better understand the nature of the collapsed wave function and conditional probability, we return to the probability experiment with Alice and Bob. The initial state (for the Ace of Hearts and the Ace of Spades) is given by

$$|\psi\rangle = |A_h\rangle |A_s\rangle + |A_s\rangle |A_h\rangle. \quad (6)$$

However, once either one of the envelopes are open, the state changes (or collapses) for the person opening the envelope into either  $|\psi\rangle = |A_h\rangle \otimes |A_s\rangle$  or  $|\psi\rangle = |A_s\rangle \otimes |A_h\rangle$ . Note this is a consequence of conditional probability theory and a natural explanation can be given in terms of moving from ignorance to knowledge, without in any way suggesting that causality has been violated. Similarly, in quantum mechanics an analogous situation arises, although the property of rotational invariance and SU(2) relativity involves purely a quantum phenomenon.

To summarize, from the perspective of this paper while the principle of superposition as prescribed by the Copenhagen convention still remains and the initial quantum state is best written as a complete set of eigenvectors, it does not follow that the act of measuring alone determines the observed eigenstate. Rather the measuring devices because of its anisotropic nature, selects one of the preexisting eigenstates as an axis of rotation, thus fixing the preassigned information along that axis, while breaking the preassigned correlated information in other directions because of the rotation effect. Moreover, the anisotropic measurement increases our knowledge of the situation in a given direction, while at the same time destroying information about the initial isotropic state. From a probability point of view, we can say that the measuring device imposes a Markov condition on the newly emerging state by disentangling the singlet. Moreover, once an observation is made, the observed measurement can be identified with the collapsed wave function in accordance with the mathematical rules for projection operators.

## 4 INEQUALITIES TO EQUALITIES

In view of the above, we now re-formulate Bell's perspective from the perspective of  $SU(2)$ relativity. In Bell's original paper, he argues that for a system of particles in the singlet state the expectation  $E(\lambda_i^{(1)}\lambda_j^{(2)})$ , identified with "hidden" parameters, is never equal to the quantum expectation  $\langle \sigma_i^{(1)}\sigma_j^{(2)} \rangle$  where the (1) and (2) refer to particle 1 and 2 respectively. We now proceed to calculate this expected value,  $E(\lambda_i^{(1)}\lambda_j^{(2)})$ , of the spin measured on a coupled system of particles, in the directions  $\vec{n}_i$  and  $\vec{n}_j$ . Recall  $\vec{n}_i \cdot \vec{n}_j = \cos \theta_{ij}$ . Let  $P(\lambda_i^{(1)}, \lambda_j^{(2)})$  be the joint distribution function associated with the two directions of measurement then since the particles are in the singlet state

$$E(\lambda_i^{(1)}\lambda_j^{(2)}) = \sum \lambda_i^{(1)}\lambda_j^{(2)} P(\lambda_i^{(1)}, \lambda_j^{(2)}) \quad (7)$$

$$= \sum \lambda_i^{(1)} P_1(\lambda_i^{(1)}) \lambda_j^{(2)} P_{ji}(\lambda_j^{(2)} | \lambda_i^{(1)}) \quad (8)$$

$$= (1)(\frac{1}{2})(-1) \cos^2(\frac{\theta_{ij}}{2}) + (-1)(\frac{1}{2})(1) \cos^2(\frac{\theta_{ij}}{2}) \\ + (1)(\frac{1}{2})(1) \sin^2(\frac{\theta_{ij}}{2}) + (1)(\frac{1}{2})(-1) \sin^2(\frac{\theta_{ij}}{2}) \quad (9)$$

$$= -\cos^2 \frac{\theta_{ij}}{2} + \sin^2 \frac{\theta_{ij}}{2} \quad (10)$$

$$= -\cos \theta_{ij} \quad (11)$$

$$= -\vec{n}_i \cdot \vec{n}_j \quad (12)$$

$$= \langle \sigma_i^{(1)} \sigma_j^{(2)} \rangle. \quad (13)$$

Note our equation (7) is identical in form to equation (2) of Bell's paper while our equation (12) and (13) are identical to equation (3) of Bell's paper. In our case, however, both equations (7), (12) and (13) coincide while in Bell's paper they do not. This difference stems from the fact that Bell does not prioritize the direction of the measuring magnetic field when dealing with hidden parameters and his derivation presupposes that three independent measurements can be made in three different and arbitrary directions without consideration of the  $SU(2)$  reference frame. Wigner's derivation of a comparable inequality to that of Bell's rests on the same fallacy [11].

## 5 CONCLUSION

In conclusion, note that the epistemological model presented in this paper presents no difficulty in assigning a real objective state to the elementary particle. However, a numerical assignment of values to this objective state should

- (1) take into account the  $SU(2)$  relativity of the situation,
- (2) realize that such a state can be measured,
- (3) recognize that the measurement changes the system by redefining the initial

conditions (the uncertainty principle),

(4) understand the limited way in which the singlet state allows us to take a second measurement of the initial system.

This last point takes us back to Einstein's original version of the EPR paradox which was formulated in terms of position and momentum and not spin [3]. From the perspective of this paper, Einstein was partially correct. Realism still exists in nature and the projection postulate of QM, if viewed from the (weak) Copenhagen perspective, violates this realism. It is unfortunate, therefore, that the measurement of spin itself has become the standard for testing the paradox. It seems to me that in reality the Alain Aspect [1] experiment and other such experiments challenge the naive realist interpretation of quantum mechanics, but they do not prove Einstein's position to be incorrect. Naive realism is not realism, it is just a flawed model of reality. SU(2) relativity not only removes the inconsistency associated with naive realism by preserving locality, but also seriously challenges the "action at a distance" associated with the weak Copenhagen interpretation. Indeed, from this perspective Einstein's objection is still valid.

On the other hand, Einstein's rejection of the statistical basis of quantum mechanics stemming from the (weak) Copenhagen interpretation cannot, in my opinion, be fully justified. A rejection of the projection postulate cannot be used to reject a statistical interpretation of quantum mechanics. In the above model, the quantum state actually exists in an ontological way, as an element of physical reality. It results from more than just our lack of knowledge of initial conditions, as suggested by the ensemble interpretation associated with Einstein. <sup>1</sup>Indeed, the dynamical properties of spin are not only affected by the measuring process, as exemplified in the case of the spin-singlet state becoming disentangled, but they do so in such a way as to maintain realism while avoiding the difficulties associated with Bell's inequality.

Specifically, SU(2) relativity combined with conditional probability theory and the projection postulate, not only guarantee realism but also the necessity of a statistical interpretation of physical reality, associated with Heisenberg's uncertainty principle. In essence the uncertainty relations are a proof that a complete set of initial conditions can never in principle be fully specified, because the act of measurement itself changes the original system. It is precisely this inability to specify precise initial conditions that allows a probability theory to emerge. In this context, Lindsey and Margenau note that the principle of indeterminacy in QM "is much more thoroughgoing, for it converts a question of convenience into a matter of necessity. It expresses the conviction ultimately that we shall never in any case be able to carry out even in principle the measurement necessary for the exact determination of boundary conditions in any physical problem" [2]. It then follows from this that "ignorance of them [boundary conditions] forces us back on probability considerations" [2].

Finally on a philosophical level we have modified the weak Copenhagen

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<sup>1</sup>Although, it should be pointed out that the (weak) interpretation, is also subjected to the law of large numbers and can also be viewed from the ensemble perspective.

interpretation of quantum mechanics with regards to its interpretation of the projection postulate and its failure to grasp  $SU(2)$  relativity. Our interpretation is based on a critical realist approach which takes the given data to be objective, without falling into some Newtonian dualistic interpretation of reality. We do not put the observer outside of the world he is observing as is the approach in classical mechanics or hidden variable theory (naive realism being a case in point), nor do we go to the other extreme of giving a singular importance to the observer as in the Copenhagen interpretation. The measurement of spin is itself a  $SU(2)$  relativity effect. We interact with the physical world because we are part of the physical world and indeed it is precisely this interaction that permits us to do physics. The laws of interaction are part of the objective laws of the universe and are there to be discovered. Quantum physics has tried to incorporate those interactions pertaining to physics into its axioms. To the extent that it has succeeded, quantum mechanics gives a more real and objective picture of physical reality, the uncertainty relations being a case in point. To the extent that these axioms are not yet complete (the projection postulate), quantum mechanics should be willing to modify its basic axioms in the light of the evidence.

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